Optica Applicata, Vol. XXXIII, No. 2–3, 2003

Testing of plano-optical elements

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The most common criteria of the quality of optical surfaces and elements are analysed in detail in this paper. Both geometrical-optics criteria and diffraction criteria are defined. The connection between different criteria is also described. Some terms lacking classical analogy in optics are described, *e.g.*, the focal length of the plane parallel plate, prism, *etc*.

Keywords: optical testing, aberrations, optical imaging.

1. Introduction

The present technology of fabrication of optical elements with plane surfaces, *i.e.*, prisms, plane-parallel plates, mirrors, *etc*., does not allow making them absolutely perfect and therefore these elements affects the wavefront aberration. Other source of aberrations of such optical elements are defects in materials from which individual optical elements are fabricated, *e.g.*, inhomogeneities of optical glass. These two categories are called technological or manufacturing aberrations [1].

Any optical system designed is composed of several optical elements that must be made within defined tolerances to guarantee the required imaging properties of the optical system. In practice, optical elements can be tested using various measuring techniques [2]. Use of some specific testing technique depends on characteristics of the optical part tested (prism, plane-parallel plate, mirror, *etc*.) and the range of allowable tolerances for the size, quality of optical surfaces, transmission, *etc*. Now, we focus only on geometrical tolerances of the shape of optical surfaces and imaging properties of the optical element under test.

The simplest methods for checking the geometrical shape of optical surface are mechanical measuring methods [2]. These techniques have many disadvantages, *e.g.*, the possibility of measuring the shape of optical surface only on a discrete set of points situated on the surface, the risk of damaging the surface with the measuring instrument, a long time needed for measurement, *etc*. Because of the above drawbacks these methods are used seldom, especially for testing optical elements of lower quality or in piece production of special optical parts. A typical representative of measuring instruments based on mechanical testing methods is a coordinate measuring machine that makes it possible to measure the deviation of the measured "plane" optical surface from an ideal plane surface with the accuracy better than 1 μm.

For very precise measurements of optical surfaces techniques based on the principle of interference or diffraction of light are used [3]. The simplest optical method for measuring deviations of the optical surface tested from the nominal surface is a comparison of the surface under test with the calibrated optical surface that is made with an order higher accuracy than the surface tested. This method is based on the interference principle. The calibrated optical surface is very carefully placed close to the measured surface and interference fringes are observed. From the shape of these interference fringes a quality of the measured optical surface can be determined [2], [4]–[6]. An experienced optical engineer in the optical production can estimate visually deviations from an ideal shape of the surface tested up to $\lambda/10$, where λ is the wavelength of light used in the control process. However, fabrication of calibrated optical surfaces for different types of optical elements is very difficult and expensive. Further, both calibrated and tested surfaces can be damaged by scraping during the measurement process. It is evident that there was a big effort put into a development of noncontact and fully automatic evaluation of the shape of fabricated optical elements. The noncontact interferometric testing technique can be implemented by properly designed interferometers, either Fizeau or Twyman–Green type [2], [7]–[9]. With the present development of solid-state array sensors, *e.g.* CCD, and other optoelectronic elements the evaluation of the measurement using above-mentioned interferometers became fully automatic.

An arbitrary shape of optical surface (flat, spherical or aspherical) can be tested with various types of appropriate phase measuring procedures [4], [6], [10], [11]. These techniques determine the phase of the wave field under investigation from the measurements of the intensity of interference field that arises from the interference of the tested and reference wave fields in interferometric testing [7]. The accuracy obtained with the above-mentioned interferometric systems is from $\lambda/20$ up to $\lambda/100$ that is suitable for most requirements on the measuring accuracy in optical industry.

Another type of methods for controlling the properties of plano-optical elements are noncontact geometrical optical methods [2]. Using these testing techniques angle deviations of plane optical surfaces can be determined with the accuracy of a fraction of an angle second. These methods are widely used in optical industry for their accuracy, objective approach and relative simplicity. In the following text we will focus on the measurement process and analysis of the quality of plano-optical elements.

2. Criteria of quality of optical elements with plane surfaces

The quality of an optical surface or an optical element can be described with various characteristics. The basic quality criteria include:

– *For plane surfaces*: the geometrical deviation of the surface shape from an ideal flat surface, the number and shape of interference fringes, the focal length of the plane surface, the astigmatism of the plane surface, the Strehl definition of the surface, and the resolving power of the surface.

– *For plane parallel plates*: the deformation of the transmitted wavefront, the focal length of the plate, the Strehl definition of the plate, the resolving power of the plate, and the deviation from parallelism.

– *For prisms*: the deformation of the transmitted wavefront, the focal length of the prism, the Strehl definition of the prism, the resolving power of the prism, and the deviation from parallelism.

Before making a detailed theoretical analysis of individual criteria for testing the quality of plano-optics, let us recall in short the geometrical and diffraction theory of optical imaging.

2.1. Geometrical theory of optical imaging

Firstly, we describe briefly the theory of optical imaging from the viewpoint of geometrical optics, *i.e.*, we do not consider wave properties of light. Aberrations of optical systems are deviations in imaging properties of real and ideal optical systems. Aberrations of real optical systems are caused by several factors, especially by refraction and reflection of light on surfaces of the optical system, or imperfections during fabrication and defects of materials from which optical elements are made.

Figure 1 shows an ideal optical system with its ideal imaging properties. Consider, *e.g.*, an off-axis point B, from which a spherical wavefront Σ propagates. Due to the fact that an ideal optical system images a point in the object plane η as a point in the image plane η' the spherical wavefront Σ will be after passing through the ideal optical system transformed again into a spherical wave Σ_0' with the centre at the point B' that is a Gaussian image of the point B. Rays propagating from the point B will intersect the image plane η' at the point B' after passing through the optical system.

Let us consider the case of imaging properties of a real optical system, *i.e.*, the system with aberrations, as shown in Fig. 2. Consider again some off-axis point B from

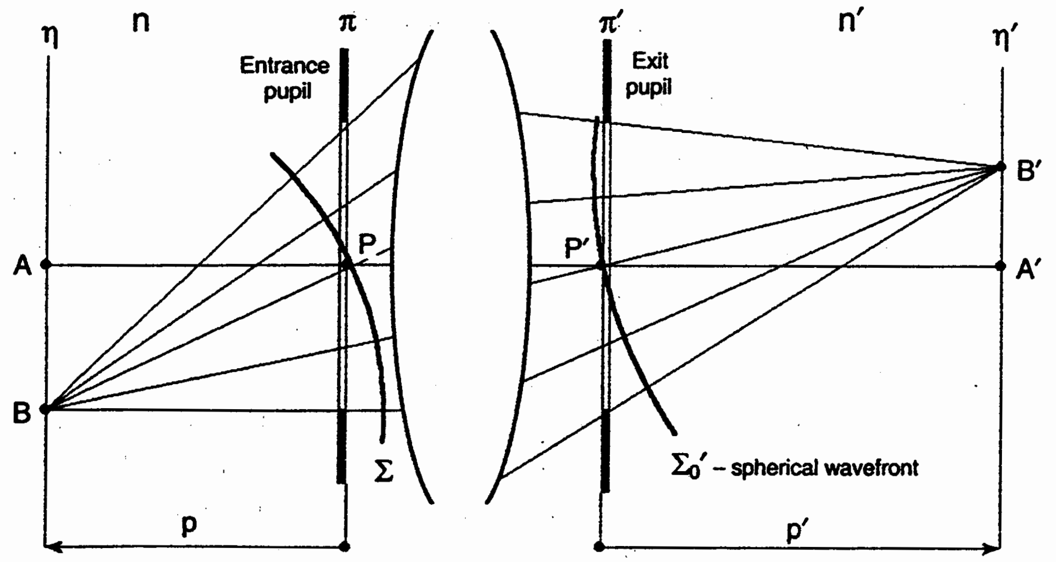

Fig. 1. Imaging properties of an ideal optical system.

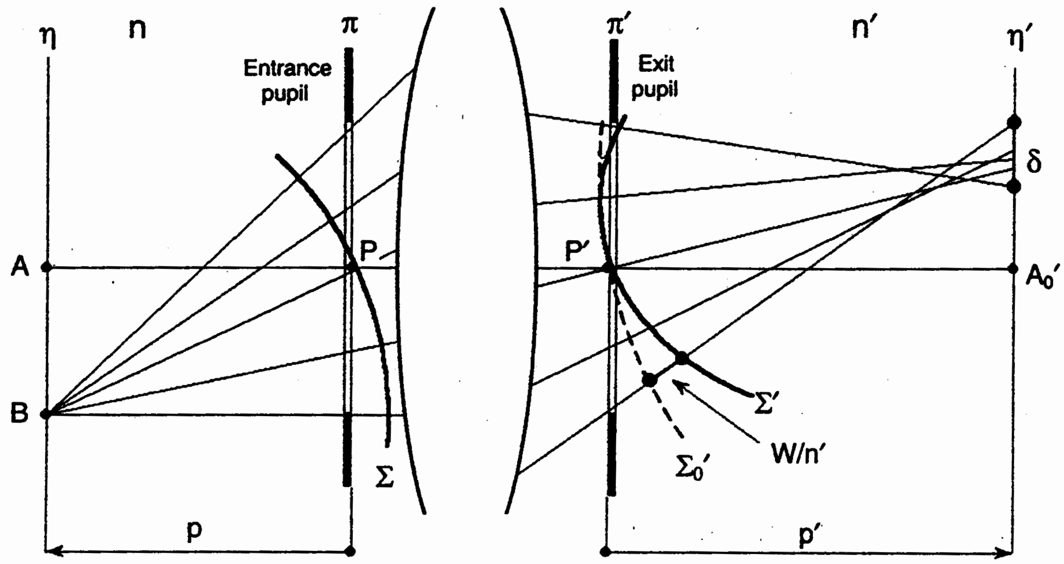

Fig. 2. Imaging properties of a real optical system (δ – diffraction disc, W – wave aberration).

which a spherical wave propagates. After passing through the real optical system the spherical wavefront Σ is transformed into a wavefront Σ' of a general shape. The deviation W of the wavefront Σ' from the ideal spherical wavefront Σ_0' is called a wave aberration of the optical system. Rays propagating from the point B will not intersect the image plane at one point after passing through the optical system. These rays will intersect the image plane in a diffraction disc. Increasing the wavefront aberration the size of the diffraction disc δ will also increase. A position of the image plane η' is chosen so that the ideal image A_0' of the point A is lying in the image plane.

To obtain the best imaging properties of the real optical system the wave aberration must be corrected. The remaining value of the aberration must be reduced in the widest spectral region. If the uncorrected wave aberration W of the optical system is lower than a quarter of the wavelength of light λ, *i.e.* $W < \lambda/4$, for all points of the object, then the image of the real optical system is not practically distinguishable from the image of the perfect optical system without any aberration for a given wavelength λ (sometimes it is called Rayleigh's quarter wavelength rule) [9].

2.2. Diffraction theory of optical imaging

In the preceding text we ignored wave aspects of light and described the geometrical theory of optical imaging. In geometrical optics an ideal optical system, *i.e.*, optical system without any aberration, images a point in the object plane into a point in the image plane. Considering wave properties of light and the finite size of optical systems the image of a point in the object plane will be the diffraction pattern in the image plane. The specific distribution of energy within the diffraction pattern depends on the wavelength of light, shape of the pupil, *f*-number and aberrations of the optical system.

An optical system whose imaging properties are limited only by the wave character of light, *i.e.*, system is without aberrations, is called the diffraction limited optical system. In Figure 3, a situation of imaging an axis point A and off-axis point B by the optical system is shown. Due to the wave character of light their images A' and B' will

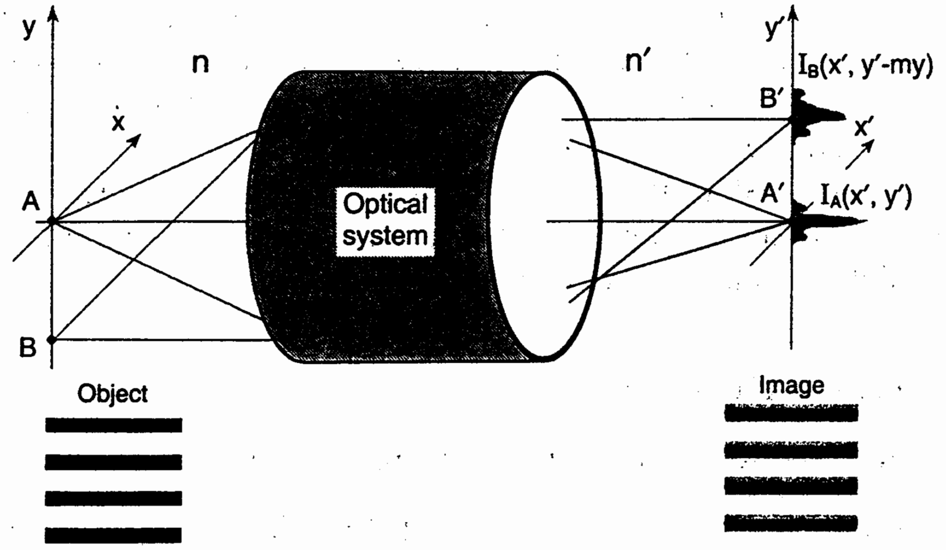

Fig. 3. Point spread function.

be diffraction patterns with distributions of the intensity of light $I_A(r)$ and $I_B(r)$ in the image plane.

The response of the optical system to a point signal is called the point spread function (PSF). The shape of the PSF, *i.e.*, the energy distribution in the diffraction pattern, depends on a position of the imaged point in the object plane and on the distance of the object plane from the optical system [1], [3].

Assume now that the image will be formed in the incoherent light, *i.e.*, daylight, and the optical system will be diffraction limited. Further suppose that the entrance pupil of the optical system is circular, equally illuminated and has constant transmission properties. This case mostly occurs in practice and the point spread function of such an optical system is then given by [3], [8], [9]:

$$I(r) = I_0\left[\frac{2J_1(a)}{a}\right]^2 \qquad (1)$$

where I_0 is the intensity in the centre of the diffraction pattern, $J_1(a)$ – the Bessel function of the first order with the argument $a = \pi r/\lambda c$ (λ – the wavelength of light, c – the *f*-number of the optical system and r – the distance from the centre of the diffraction pattern). The Bessel function $J_1(a)$ has first zero value for $r = r_A = 1.22\lambda c$. The quantity r_A is called a radius of Airy disc. The diameter of the central part of the diffraction pattern can be expressed as

$$d_A = 2.44\lambda c \qquad (2)$$

and is called the Airy disc. The surface of imaged objects can have a very complex pattern and its individual parts can differ in the refinement and contrast of the pattern. The optical system is not able to image individual patterns of the object with the same contrast. The image will always have worse contrast than the object. The finest (high frequency) patterns will be imaged with lower contrast than coarse (low frequency) patterns. It happens very often that some very high frequency patterns of the object cannot be imaged by a given optical system at all. In the case of small aberrations, for evaluation of the quality of optical systems we use the Strehl definition (Strehl ratio) [3], [9] that is defined as the ratio between the maximum of the PSF of the real optical system and the maximum of the PSF of the diffraction limited system. With respect to the Strehl definition we consider the optical system to be equivalent to the ideal optical system if the Strehl definition is higher than 0.8.

3. Testing of plano-optical elements

3.1. Testing of plane optical surfaces using Fizeau interferometer

A principal scheme of the Fizeau interferometer [2], [7]–[9] for the testing of plane optical surfaces is shown in Fig. 4. The light from the source S (mostly laser) passes through the semitransparent mirror M and the objective O_1. The last surface of the objective O_1 is a very precise plane surface that is called the reference plane surface. All optical surfaces of the objective O_1 with the exception of the reference surface are coated with antireflective layers. The reflectivity of the reference surface is approximately 4% and a part of the incident light is therefore reflected and this reflected light forms a reference wavefront. The remaining part of incident light is transmitted through the objective O_1 and is reflected backwards from the tested optical "plane" surface.

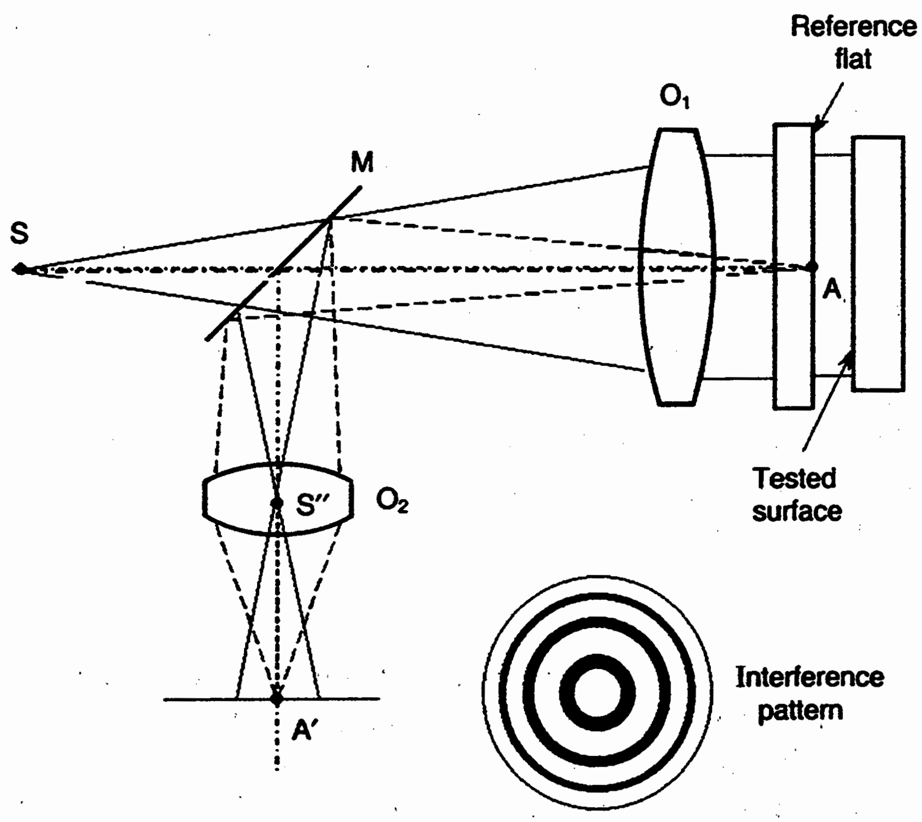

Fig. 4. Fizeau interferometer.

The object wavefront reflected from the surface tested interfere with the reference wavefront and the interference field is then localized on the reference surface. This interference field is then imaged by the objective O_2 onto the detector plane, *i.e.*, an arbitrary point A of the interference field is transformed by the objective O_2 onto a point A' in the detector plane. The entrance pupil of the objective O_2 is located in a place where the image S'' of the source S after reflection on the reference surface of the objective O_1 is formed. The detector, *e.g.*, CCD sensor, then detects the distribution of the intensity of the interference field (interference pattern). From one or more captured interference patterns we are able to determine the deformation of the surface tested. There exist several techniques for automatic evaluation of the phase distribution of the interference field that enable us to determine the shape of the optical element being tested [7].

Consider now that recorded interference fringes are circular. If D is the diameter of the plane surface under examination, N – the number of interference fringes and λ – the wavelength of light, then for the radius of curvature of the plane surface it holds that ($\lambda = 555$ nm)

$$r_{\text{plan}} = \frac{D^2}{4\lambda N} \approx \frac{450D^2}{N}. \tag{3}$$

3.2. Measurement of angle deviations using autocollimator

During fabrication of prisms and plane-parallel plates some deviations from required parameters of these optical elements can occur. In practice it is often needed to determine angle deviations of single surfaces of plane-parallel plates. These measurements can be most easily performed using an autocollimator [2]. The basic principle of testing optical elements with the autocollimator is shown schematically in Fig. 5.

The autocollimator consists of the object lens O, plates A and C with observing patterns, the semi-transparent plate B and the source of light L that illuminates the

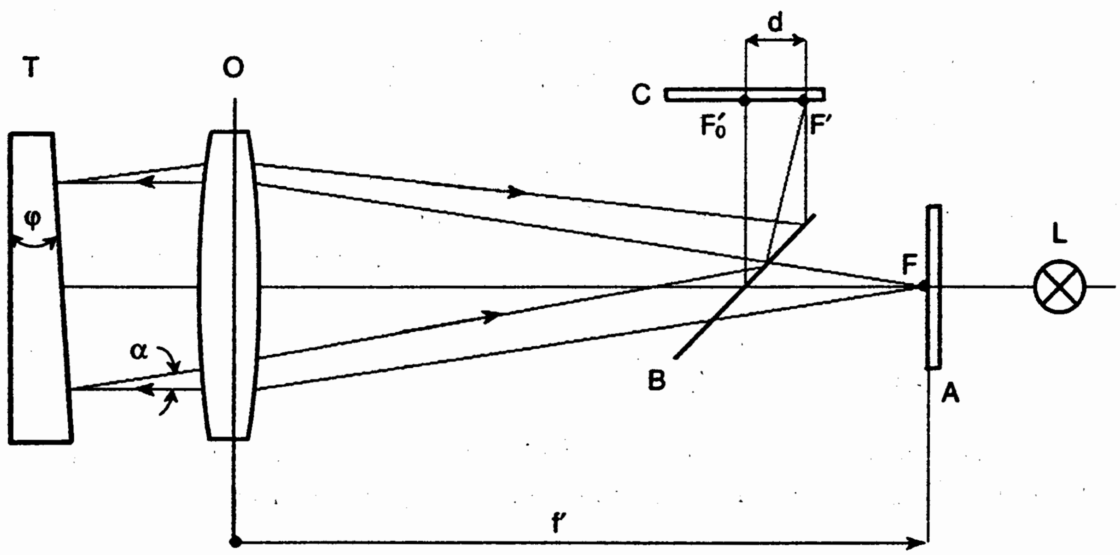

Fig. 5. Principle of the autocollimator.

plate A. Plates A and C are placed into a focus of the object lens O and the patterns are usually a reticle on the plate A and two perpendicular linear measuring scales on the plate C. In front of the object lens the tested object T, *e.g.*, a plane-parallel plate, is placed and we observe either with an eyepiece or a CCD camera the position of the image F' of the reticle centre (point F). If α is the angle between the incident and reflected rays, then for the displacement of image F' from the nominal image position F_0' we have

$$d = f'\tan\alpha \tag{4}$$

where f' is the focal length of the object lens of autocollimator. In the case the object T under test is a wedge with an angle φ that is made from glass with the refractive index n, then the angle α can be expressed as $\alpha = 2n\varphi$. In another case of measuring a plane mirror tilted with respect to the axis of the autocollimator at angle φ, for the angle α holds: $\alpha = 2\varphi$.

4. Analysis of basic criteria of the quality of plano-optical elements

Let us focus in more detail on individual optical elements and criteria describing their optical quality. Plano-optical elements, *i.e.*, mirrors, plane-parallel plates and prisms, are very often used in various optical systems in practice. Due to a technological process of fabrication the real shape is little changed from the nominal shape of the optical elements. It has one negative effect that such optical elements then cause wavefront aberrations in the imaging process and it leads to a decrease in imaging quality of optical systems, where these optical components are mounted. From the theory of geometrical optics the focal length of ideal plano-optical elements is infinite. Due to deformation of their optical surfaces during fabrication real plano-optical components have the finite focal length and they are affected by various types of wave aberrations. We can evaluate the quality of plano-optical elements in the same way as the quality of imaging properties of classical optical (lens) systems is determined.

We will now describe, using several examples of different plano-optical elements, how to determine basic criteria of their quality, *e.g.*, the focal length, *etc*. A basic assumption in our description is that deviations of plano-optical surfaces and elements from their nominal values are very small to ensure required imaging properties of designed optical systems consisting of these optical components.

With respect to Eq. (3) we obtain for the focal length of the refractive plane surface

$$f'_{\text{refraction}} = \frac{D^2}{4\lambda(n-1)N} \approx \frac{450D^2}{(n-1)N} \tag{5}$$

where the plane surface of diameter D is made from glass with the refractive index n and N is the number of interference fringes observed during interferometric testing of the surface. In the case of the reflective plano-optical surface, *i.e.*, plane mirror, it holds for the focal length

$$f'_{\text{reflection}} = \frac{D^2}{8\lambda N} \approx \frac{225 D^2}{N}. \tag{6}$$

The wave aberration of the plane surface is then given by

$$W_{\text{surface}} = \frac{D^2}{8 f'_{\text{surface}}}. \tag{7}$$

In the case of refractive plane surface we substitute $f'_{\text{surface}} = f'_{\text{refraction}}$ into the preceding equation. In the case of reflective plane surface, $f'_{\text{surface}} = f'_{\text{reflection}}$ must be substituted into Eq. (7).

If the plane reflective surface, *i.e.*, plane mirror, is illuminated at incidence angle ε, then the reflected wavefront is astigmatic. Using Eq. (3) and Abbe–Young relations [9] for tracing of an astigmatic beam through the optical system we obtain the following relation for the astigmatism δ_{astig} of the plano-optical surface

$$\delta_{\text{astig}} = \frac{D^2}{8\lambda N} \sin\varepsilon \, \tan\varepsilon \approx \frac{225 D^2}{N} \sin\varepsilon \, \tan\varepsilon. \tag{8}$$

For the wave aberration we obtain

$$W_{\text{astig}} = \frac{\delta_{\text{astig}}}{16 c^2_{\text{surface}}} = \frac{\lambda}{2} N \sin\varepsilon \, \tan\varepsilon \tag{9}$$

where

$$c_{\text{surface}} = \frac{f'_{\text{reflection}}}{D} \tag{10}$$

is the *f*-number of the surface. Then the Strehl definition can be derived from

$$\text{SD} = 1 - \left(\frac{2\pi}{\lambda}\right)^2 \frac{W^2_{\text{astig}}}{6} = 1 - \frac{\pi^2}{6} (N \sin\varepsilon \, \tan\varepsilon)^2. \tag{11}$$

In practice the most usual case is $\varepsilon = 45°$. Then the wave aberration and the Strehl definition can be obtained from

$$W_{\text{astig}} = 0.35 \lambda N, \tag{12}$$

$$\text{SD} = 1 - \frac{\pi^2}{12} N^2 \approx 1 - 0.82 N^2. \tag{13}$$

In the case of a plane-parallel plate with diameter D made from glass with the refractive index n, where the first refractive surface has N_1 interference fringes and the second surface has N_2 interference fringes, the focal length f'_{plate} of the plane -parallel plate is given by

$$f'_{\text{plate}} = \frac{D^2}{4\lambda(n-1)(N_1-N_2)} \approx \frac{450D^2}{(n-1)(N_1-N_2)}. \tag{14}$$

The following sign convention from the theory of geometrical optics is used for determining the signs of N_1 and N_2. If the first surface is convex, then the sign of N_1 is positive. In the case of the concave surface the sign of N_1 is negative. The sign convention of the second optical surface is inverse. The f-number of the plane parallel plate is then

$$c_{\text{plate}} = \frac{f'_{\text{plate}}}{D}. \tag{15}$$

The diameter of the Airy disc of the plane-parallel plate can be expressed as

$$d_A = 2.44\lambda c_{\text{plate}} = \frac{0.61D}{(n-1)(N_1-N_2)} \tag{16}$$

and the wave aberration of the plane-parallel plate is given by

$$W_{\text{plate}} = \frac{D^2}{8f'_{\text{plate}}} = \frac{\lambda}{2}(n-1)(N_1-N_2). \tag{17}$$

For the Strehl definition of the plane-parallel plate we obtain

$$\text{SD} = 1-\left(\frac{\pi}{\lambda}\right)^2\frac{W^2_{\text{plate}}}{3} = 1-\frac{\pi^2}{12}(n-1)^2(N_1-N_2)^2. \tag{18}$$

Let us now consider the lateral chromatic aberration that is associated with the error in parallelism of the plane-parallel plates and prisms. If φ is the angle error in parallelism of the plane-parallel plate or prism, then the deviation of the light beam that propagates through these optical elements is $\delta = (n-1)\varphi$, where n is the refractive index of glass from which the optical element is made. Due to change of the wavelength λ the angle deviation δ is also changed. This change is given by

$$\Delta\delta_\lambda = \Delta n\varphi \tag{19}$$

where Δn is the change of the refractive index n due to the change of the wavelength of light λ. If D is the diameter of the optical element being tested, then for the wavefront aberration the following holds

$$\Delta W_\lambda = \Delta\delta_\lambda/D. \tag{20}$$

For example, if a nonparallel plate or prism is placed in front of the optical system with the focal length f', then for the lateral chromatic aberration $\delta Y_B'$ in the image plane of the optical system we obtain

$$\delta Y_B' = \Delta \delta_\lambda f'. \tag{21}$$

The modulation transfer function in the case of lateral chromatic aberration is given by [3]

$$M(R) = M_0(R) \frac{\sin(\pi R \delta Y_B')}{\pi R \delta Y_B'} \tag{22}$$

where R denotes the spatial frequency (lines/mm), $\delta Y_B'$ is the lateral chromatic aberration and M_0 is the modulation transfer function of the diffraction limited optical system. From the preceding relation it is clear that for the spatial frequency

$$R = 1/\delta Y_B' \tag{23}$$

we will have $M = 0$. The lateral chromatic aberration must reflect some condition if the quality of imaging should not be significantly decreased. This mathematical condition can be expressed as

$$\frac{\sin(\pi R \delta Y_B')}{\pi R \delta Y_B'} \geq T \tag{24}$$

where T is a chosen threshold, *e.g.*, $T = 0.8$. To determine acceptable value of the lateral chromatic aberration that fulfils the preceding condition we express the left-hand side of Eq. (24) as a Taylor series. We obtain

$$\frac{\sin(\pi R \delta Y_B')}{\pi R \delta Y_B'} \approx 1 - \frac{(\pi R \delta Y_B')^2}{6} \geq T \tag{25}$$

where we restricted the series expansion to the first two terms only. The tolerable value of the lateral chromatic aberration is then given by

$$\delta Y_B' \leq \frac{\sqrt{6(1-T)}}{\pi R} \approx 0.8 \frac{\sqrt{1-T}}{R}. \tag{26}$$

Using this relation it is possible to determine the acceptable value of the lateral chromatic aberration that fulfil the tolerance condition (24). As an example we show the acceptable value of the lateral chromatic aberration for $T = 0.8$ and the spatial frequency $R = 20$ lines/mm. We obtain

$$\delta Y_B' \leq 0.018 \text{ mm}. \tag{27}$$

5. Conclusions

In the paper, a detailed analysis of the basic technological aberrations of plano-optical elements has been made. Such terms as the focal length and the *f*-number of a plane -parallel plate have been defined and described equations for their calculation. Further relations for calculation of the wave aberration, astigmatism and Strehl definition have been derived. The influence of the lateral chromatic aberration for plano-optical elements has been investigated using the modulation transfer function and the relation for determination of a tolerable value of lateral chromatic aberration has been described.

Acknowledgment – This research was supported by grant No. 202/02/0314 from Grant Agency of the Czech Republic.

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Received October 11, 2002