Settlement of Shallow Footings

Stress Increment under Footing

Excess of Vertical Stress

Subsoil Settlement due to Stress Change
Settlement Consists of:

• Immediate settlement: $s_i$
  – No volume change of subsoil, displacements only

• Settlement caused by primary consolidation: $s_p$
  – Dissipation of pore-water pressure connected with water flow and significant volume change of the soil

• Settlement by secondary consolidation: $s_s$
  – Volume change due to secondary compression and $u=0$ – long-term development
Immediate Settlement

\[ S_i = q \cdot \frac{B}{E_u} \cdot I_u \]

\[ I_u = \mu_1 \cdot \mu_0 \]
Settlement by Primary Consolidation

• Based on:
  – Oedometer modulus and subsoil composed of individual layers,
  – Calculation of Origin Stress and Stress Increment,
  – Calculation of Settlement:

\[
s = \sum_{i=1}^{n} \frac{\sigma_{zi}}{E_{oed,i}} \cdot h_i = \sum_{i=1}^{n} \frac{\sigma_{zi} - m_i \cdot \sigma_{or,i}}{E_{oed,i}} \cdot h_i
\]
Settlement by Secondary Consolidation
Calculation of Settlement $s_s$

\[ s_s = h \cdot \alpha_s \cdot (\log t_2 - \log t_1) \]

Coefficient of Secondary Consolidation:

- Sand \( \approx 0.00003 - 0.00006 \)
- Loess soil \( \approx 0.0004 \)
- Clay \( \approx 0.001 \)

\[ \alpha_s = \frac{\Delta h}{h} \cdot \frac{1}{\log t_2 - \log t_1} \]
Boussinesq - Point load
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\[ \sigma_R = A \cdot \frac{\cos \beta}{R^2} \]

\[ s = 2\pi R \cdot \sin \beta \cdot R \cdot d\beta \]

\[ F = A \cdot 2\pi \int \sin \beta \cdot \cos^2 \beta \cdot d\beta = \frac{2}{3} \pi \cdot A \]

\[ \sigma_R = \frac{3Fz}{2\pi R^3} \]

\[ \sigma_z = \frac{3Fz^3}{2\pi R^5} \]

\[ A = \frac{3F}{2\pi} \]
V / H Stress

\[
\sigma_2 = 2 \quad \sigma_3 = 2
\]

\[
\eta_R \cdot \cos \beta \cdot a = \sigma_2 \cdot c
\]
\[
\eta_R \cdot \cos \beta \cdot c - \cos \beta = \sigma_2 \cdot c
\]
\[
\sigma_2 = \eta_R \cdot \cos^2 \beta
\]

\[
\sigma_x = 2
\]

\[
\eta_R \cdot \sin \beta \cdot a = \sigma_x \cdot c
\]
\[
\eta_R \cdot \sin \beta \cdot c \cdot \sin \beta = \sigma_x \cdot c
\]
\[
\sigma_x = \eta_R \cdot \sin^2 \beta
\]
Stress increment calculation

\[ \sigma_x = \frac{P}{\pi} \left[ \alpha - \sin \alpha \cdot \cos 2\beta' \right] \]

\[ \sigma_x = \frac{P}{\pi} \left[ \alpha + \sin \alpha \cdot \cos 2\beta' \right] \]

\[ \tau_{xz} = \frac{P}{\pi} \sin \alpha \cdot \sin 2\beta' \]

\[ \sigma_x = \frac{P}{\pi a} \left( \frac{a \beta + x \alpha + 2z \ln \frac{R_2}{R_1}}{2} \right) \]

\[ \sigma_y = \frac{2P}{\pi \tan \alpha} \left( \frac{a \beta + x \alpha + z \ln \frac{R_2}{R_1}}{2} \right) \]

\[ \sigma_z = \frac{P}{\pi a} \left( \frac{a \beta + x \alpha}{2} \right) \]

\[ \tau_{xz} = -\frac{P}{\pi a} \cdot z \alpha \]
Stress increment calculation

\[ \sigma_x = \frac{P}{\pi a} \left[ a (\beta + \beta') - b (\alpha + \alpha') + x (\alpha - \alpha') + 2z \ln \frac{R_2 R_1'}{R_1 R_1'} \right] \]

\[ \sigma_z = \frac{P}{\pi a} \left[ a (\beta + \beta') - b (\alpha + \alpha') + x (\alpha - \alpha') \right] \]

\[ \sigma_y = \frac{2P}{\pi a m} \left[ a (\beta + \beta') - b (\alpha + \alpha') + x (\alpha - \alpha') + z \ln \frac{R_2 R_1'}{R_1 R_1'} \right] \]

\[ \tau_{xz} = -\frac{P z}{\pi a} (\alpha - \alpha') \]
Stress Increment

Large Embankment Construction  Footing Depth
Stress Increment

Re-construction

Dewatering
Stress: Origin / Increment due to $V_{ds}$
Vertical Stress Distribution

Uniform loaded rectangular area – site plan

Distribution of vertical stress increment bellow indicated points - section
Contact Stress Distribution

\[ k = \frac{E}{E_{\text{def}}} \cdot \left(\frac{t}{l}\right)^3 \quad \text{If } k > 1.0 \text{ Stiff Footing} \]
Stress Increment Calculation

Circular Footing
Footing Depth – Stress Correction

Rectangular Footing

Strip Footing
Rock Base - Stress Correction
Simplified Settlement Calculation

Strip Footing: $e_1 = 1 + 0.35 \cdot \tan^{-1}(d/z)$

Square: $e_1 = 1 + 0.61 \cdot \tan^{-1}(1.55 \cdot d/z)$

\[ \sigma_2 = \sigma_f - \sigma_{ol} \]